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Verification of a model to predict the influence of particle inertia and gravity on a decaying turbulent particle-laden flow

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Abstract

In an earlier publication some of the authors presented a theoretical model for the calculation of the influence of particle inertia and gravity on the turbulence in a stationary particle-laden flow. In the present publication the model is extended for application to a decaying suspension. Also a comparison is given between predictions made with the model and experimental data (own data and data reported in the literature) on a decaying turbulent flow with particles in a water tunnel or in a wind tunnel. For most of the experiments a prediction with reasonable accuracy and an interpretation is possible by means of the model.

Keywords: Decaying turbulence; Particles; Two-way coupling; Theoretical model; Experimental verification

1. Introduction

The occurrence of particle-laden turbulent flows in nature and industrial applications is abundant. Several reviews about this topic have been published during recent years, see for instance Hetsroni (1989), Elghobashi (1994), Crowe et al. (1996) and Mashayek and Pandya (2003). It is known that when the mass loading of the particles is considerable, the so-called two-way coupling effect of the fluid on the particles and vice versa must be taken into account. This two-way coupling effect has been studied by means of direct numerical simulations, experiments, and theoretical models. A detailed review of these studies for a homogeneous particle-laden turbulent flow is given by Poelma and Ooms (2006).

Recently L'vov et al. (2003) developed a one-fluid theoretical model for a homogeneous, isotropic turbulent flow with particles, paying particular attention to the two-way coupling effect. It is based on a modified form of the Navier–Stokes equations with a wavenumber-dependent effective density of the particle-laden flow and an additional damping term representing the fluid–particle friction. The statistical model is simplified by a

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modification of the usual closure procedure based on the Richardson–Kolmogorov picture of turbulence. A differential equation for the budget of the turbulent kinetic energy is derived. For the case of a stationary turbulent particle-laden flow L'vov et al. solved this equation analytically for various limiting cases and numerically for the general case. The model successfully explains several observed features of numerical simulations.

In experiments the effect of gravity due to the difference in density between the particles and the carrier fluid is present. It causes a production of turbulence due to the net vertical movement of the particles with respect to the fluid, in case the density of the particles differs from the fluid density. This effect is not included in the model of L'vov et al. So for a proper comparison with experiments it was necessary to extend the theoretical model by including also the gravity effect. Such an extension has recently been made by Ooms and Poesio (2005). In the present publication a comparison is given between predictions made with this extended model and experimental data (own data and data given in the literature) for a decaying turbulent particle-laden flow derived from measurements in a water tunnel or wind tunnel.

2. Brief review of the theoretical model

Starting from the Navier–Stokes equations for the carrier fluid and Stokes' friction law for the particles L'vov et al. derive first the following equation of motion for a particle-laden flow (without considering turbulence production due to settling particles)

$$\rho_{\mathrm{eff}}(k)\left(\frac{\partial}{\partial t}+\gamma_{\mathrm{p}}(k)+\gamma_0(k)\right)\mathbf{u}(t,\mathbf{k})=-\mathbf{N}\{\mathbf{u},\mathbf{u}\}_{t,\mathbf{k}}+\mathbf{f}(t,\mathbf{k}). \tag{1}$$

$\mathbf{u}(t,\mathbf{k})$ represents the suspension velocity, t the time and $\mathbf{k}$ the wavevector. $\mathbf{f}(t,\mathbf{k})$ is the stirring force that creates turbulence in case a stationary suspension is studied. $\rho_{\mathrm{eff}}(k)$ is the wavenumber-dependent effective density of the particle-laden flow given by

$$\rho_{\mathrm{eff}}(k)=\rho_{\mathrm{f}}\left(1-\psi+\phi\frac{1+2\tau_{\mathrm{p}}\gamma(k)}{(1+\tau_{\mathrm{p}}\gamma(k))^2}\right), \tag{2}$$

in which ρ_{f} is the density of the carrier fluid, ψ the volume fraction of the particles in the suspension, ϕ the mass fraction of the particles, τ_{p} the particle response time and $\gamma(k)$ the frequency of a turbulent eddy with wavenumber k. $\gamma_{\mathrm{p}}(k)$ is an additional damping term representing the fluid–particle friction (described by Stokes' law)

$$\gamma_{\mathrm{p}}(k)=\frac{(\phi/\tau_{\mathrm{p}})[1+\tau_{\mathrm{p}}\gamma(k)-2\{\tau_{\mathrm{p}}\gamma(k)\}^2]/[2-2\{\tau_{\mathrm{p}}\gamma(k)\}^2]}{1-\psi+\phi[1-\tau_{\mathrm{p}}\gamma(k)]/[2-2\{\tau_{\mathrm{p}}\gamma(k)\}^2]}. \tag{3}$$

The damping term $\gamma_0(k)$ is due to the internal friction within the carrier fluid and is given by

$$\gamma_0(k)=\nu_{\mathrm{eff}}(k)k^2, \tag{4}$$

with

$$\nu_{\mathrm{eff}}(k)=\nu\rho_{\mathrm{f}}/\rho_{\mathrm{eff}}(k), \tag{5}$$

in which ν is the viscosity of the carrier fluid. $\mathbf{N}\{\mathbf{u},\mathbf{u}\}_{t,\mathbf{k}}$ is the nonlinear term. The explicit form of this term that was derived in the publication of L'vov et al. is not given here. It is not required for the simple closure procedure that was applied in the original publication and which is also used here. For the introduction of the energy flux in the applied closure procedure it is enough to use the fact that the modeled nonlinearity must be conservative. (The explicit form for the nonlinear term is needed, however, for more advanced closure procedures.)

From (1) L'vov et al. derive for the homogeneous and isotropic case the following budget equation for the spectrum of the density of turbulent kinetic energy $E_{\mathrm{s}}(t,k)$ of the suspension

$$\frac{1}{2}\frac{\partial E_{\mathrm{s}}(t,k)}{\partial t}+[\gamma_0(k)+\gamma_{\mathrm{p}}(k)]E_{\mathrm{s}}(t,k)=W(t,k)+R(t,k). \tag{6}$$

The left-hand side of this equation includes, apart from the time-dependent term, two damping terms: $\gamma_0(k)E_s(t,k)$ caused by the effective viscosity and $\gamma_p(k)E_s(t,k)$ caused by the fluid–particle friction. The right-hand side includes the source of energy $W(t,k)$ due to a possible stirring force (localized in the energy-containing interval of the spectrum) and $R(t,k)$ which is the energy redistribution term due to the interaction between turbulence eddies. Ooms and Poesio (2005) extended this equation by taking into account the gravity effect causing a turbulence production by settling particles due to a possible density difference between the particles and the carrier fluid. Their equation is given by

$$\frac{\partial E_s(t,k)}{2\partial t} + [\gamma_0(k) + \gamma_p(k)]E(t,k) = W(t,k) + R(t,k) + G(t,k), \tag{7}$$

in which $G(t,k)$ is the energy production term representing the turbulence generation due to particle settling. The physical meaning of $E_s(t,k)$ is still the same as in (6): when integrated over k it yields the total turbulent kinetic energy of the effective fluid averaged over all directions in real space. (In isotropic turbulence it is possible to express the energy spectrum in terms of one scalar, namely the absolute value k of the wavenumber. Batchelor (1953) suggested doing the same in non-isotropic but homogeneous turbulence by averaging over all directions of k-space, thus taking the mean value of the spectrum over spherical surfaces $k = \text{constant}$. Ooms and Poesio applied this approach. A derivation of (7), together with a detailed explanation of each term, is given by them.)

Using the assumption that the modeled nonlinearity is conservative the energy redistribution term can be written as

$$R(t,k) = -\frac{\partial \epsilon(t,k)}{\partial k}, \tag{8}$$

in which $\epsilon(t,k)$ is the energy flux through the turbulence eddies of the suspension (see L'vov et al.).

In order to be able to solve (7), some terms need to be modeled. A simple closure relation is applied for $E_s(t,k)$ in terms of the energy flux through the eddies $\epsilon(t,k)$, the effective density $\rho_{\text{eff}}(t,k)$ of the particle-laden flow and the wavenumber k. Applying dimensional analysis the following relation is found for the density of turbulent kinetic energy in terms of the energy flux

$$E_s(t,k) = C_1[\epsilon^2(t,k)\rho_{\text{eff}}(t,k)/k^5]^{1/3}. \tag{9}$$

C_1 is a constant of order unity. The inverse lifetime (frequency) of eddies $\gamma(t,k)$ (present in the expressions (2) and (3) for ρ_{eff} and γ_p) is determined by their viscous damping and by the energy loss in the cascade process

$$\gamma(t,k) = \gamma_0(t,k) + \gamma_c(t,k). \tag{10}$$

The inverse lifetime due to viscous damping has already been introduced by (4) and (5). Applying dimensional analysis, the inverse lifetime of a k-eddy due to energy loss in the cascade process is given by

$$\gamma_c(t,k) = C_2[k^2\epsilon(t,k)/\rho_{\text{eff}}(t,k)]^{1/3}, \tag{11}$$

C_2 is a again a constant of order unity.

For the modeling of the gravity term $G(t,k)$ some of the results of Faeth and co-workers are used. In a series of publications, see for instance Parthasarathy and Faeth (1990) and Mizukami et al. (1992), they studied the homogeneous turbulence generated by uniform fluxes of round glass beads falling through stagnant (in the mean) water or air. Their measurements included mean and fluctuating velocities, as well as temporal spectra and spatial correlations of velocity fluctuations. An analysis of the flow was undertaken in order to help interpret the measurements. In this analysis they explicitly consider the flow field around the particles. This flow field included the following contributions: the near-field around a particle and the particle wake. The wake flow was split into the mean flow and the turbulent flow fluctuations. It was shown that the summed contributions of all the near-field particle flows to the turbulence generated at an arbitrary point inside the carrier fluid are negligible. Summing the wake flow contributions of all the particles yielded an expression for the turbulence of the carrier fluid. For details about this interesting analysis we refer to the relevant publications mentioned before. Perhaps the most surprising feature of the calculation is the large range of frequencies (and wave numbers) in the calculated turbulence spectra. There is a significant level of turbulent energy at very low frequencies (very small wave numbers), even though the flow field is produced by small particles having large inter-particle spacings. The reason for this large range of frequencies (wave numbers), as well as the presence of appreciable energy at low frequencies (small wave numbers), is that the flow includes contributions from both the mean and turbulent velocity fields of the particle wakes. The mean velocity field of the wakes enhance the turbulent energy at low frequencies. Their contributions to the turbulence generated at an arbitrary point in the carrier fluid cannot be separated from the contributions of the turbulence in the particle wakes, since the particle distribution in the flow field is random. In spite of numerous simplifications the theory of Faeth and co-workers was helpful for explaining many features of their measurements, and the predictions were generally reasonable.

As mentioned, for our theoretical method we apply some results of Faeth and co-workers to describe the turbulence generated by the settling of particles in a decaying turbulent flow downstream of a grid in a wind tunnel or water tunnel. All the work carried out by the particles is used to generate turbulence, which yields the following expression for the total turbulence production $\rho_{\mathrm{f}}\phi v_{\mathrm{tv}}^2/\tau_{\mathrm{p}}$ in which v_{tv} is the settling velocity of the particles. We assume the following expression for the spectrum of the turbulence generated by the settling of the particles

$$G(k,t)=\frac{\rho_{\mathrm{f}}\phi v_{\mathrm{tv}}^2}{\tau_{\mathrm{p}}}\left[\frac{1}{\sigma(2\pi)^{1/2}}\exp\left(\frac{-\frac{1}{2}(k-2\pi/\Lambda)^2}{(\sigma)^2}\right)\right] \tag{12}$$

in which $2\pi/\Lambda$ and σ are respectively the mean wave number and width of the turbulence spectrum in wave number space. For Λ we use the expression for the integral length scale given by Mizukami et al. It is included in the solution procedure given in the Appendix.

The solution procedure for (7) for the case of a stationary turbulent flow with particles is described in detail by Ooms and Poesio. We have extended the theory for application to a decaying turbulent flow. The new solution procedure for the case of a decaying turbulent flow with particles is described in the Appendix. After introducing dimensionless quantities it is shown how the decay of the (dimensionless) turbulent energy spectrum of the fluid $E_{\mathrm{f}}(\tau,\kappa)$ can be calculated as function of dimensionless wavenumber κ and dimensionless time τ for certain chosen values of six dimensionless groups, namely

- ψ the particle volume fraction
- ϕ the particle mass fraction
- $\delta=\tau_{\mathrm{p}}/\tau_{\mathrm{L}}$ the dimensionless particle response time
- $Re_{\mathrm{f}}=u_{\mathrm{L}}L/v$ the fluid Reynolds number
- $Fr=\rho_{\mathrm{p}}u_{\mathrm{L}}^2/\Delta\rho gL$ the Froude number
- Λ/L the ratio of the integral length scale of turbulence generated by particle settling and the integral length scale of the grid-generated turbulence.

$\kappa=k/L$ and $\tau=t/\tau_{\mathrm{L}}$ in which L is the integral length scale of the fluid turbulence at $t=0$ and τ_{L} the integral time scale at $t=0$. u_{L} is the integral velocity scale of the fluid turbulence at $t=0$. $\Delta\rho$ is the difference in density between the particles and the fluid and g the acceleration due to gravity. The dimensionless energy spectrum $E_{\mathrm{f}}(\tau,\kappa)$ yields after integration with respect to κ the turbulent kinetic energy of the carrier fluid as function of time, which in this publication is compared with experimental data. In the comparison with experiments we will always deal with particles heavier than the fluid and so the particles settle in the fluid and generate turbulence. We will refer to this as: the gravity effect. The model shows that the gravity effect is determined by the following two dimensionless groups: Λ/L and the combination $\phi\delta/Fr^2$. $\phi\delta/Fr^2$ indicates the strength of the turbulence generated by the particles due to their settling in the carrier fluid with respect to the (decaying) turbulence generated by the grid at the entrance to the tunnel. Λ/L indicates the position in wave number space where the turbulence production takes place.

3. Comparison with experiments

We compared model predictions with experimental results, derived in a water tunnel or wind tunnel, by Poelma et al. (2006, 2007), Geiss et al. (2004) and Schreck and Kleis (1993). In our calculations we started with the case of decaying turbulence without particles. To that purpose we estimated the turbulent Reynolds number Re_f at the start of the decay process from the data given by the above authors and computed the turbulence decay. To be able to compare model predictions with the experimental results we calculated the distance x traveled by the turbulent flow downstream of the grid in the water tunnel or wind tunnel from the time t in the model by means of the following expression $x = Ut$, in which U is the average velocity in the tunnel. Thereafter we estimated the values of the other parameters at the start of the decay process and calculated the turbulence decay for the case with particles. By comparison with the result for the case without particles the influence of the particles on the turbulence decay process could be studied. This calculated influence was finally compared with the experimentally determined particle influence. The calculations for a turbulent fluid with particles were carried out in two steps; first without the gravity effect and then with the gravity effect included. In this way we were able to study in particular the importance of turbulence production due to settling of the particles on the decay process.

Poelma et al. carried out water tunnel experiments, for instance, with glass particles with different diameters and mass loadings. First we compare model predictions with experiments using glass particles with a diameter of 254 μm and for two values of the mass loading $\phi = 0.0018$ and $\phi = 0.0065$. In Fig. 1 the comparison is given for the low mass loading case. In the figure the normalized inverse value of the turbulent kinetic energy $u_0'^2/u'^2$ is given as function of the (dimensionless) distance x/M downstream of the grid. u'^2 is the turbulent kinetic energy. It has been normalized by its value $u_0'^2$ at the point $x/M = 15$ downstream of the grid (both for the experimental data and the model predictions). M is the mesh width of the grid used in the experiments. The model predictions show a somewhat upward-curving dependence of the turbulence damping on the downstream distance, whereas the experiments show the well-known linear behavior. This is likely due to the rather simple closure approximation that we have applied in our theoretical model. It may be expected that more sophisticated closure relationships will lead to improved predictions in this respect.

Before discussing the results shown in Fig. 1 we explain, why the results in this figure (and also the next ones) are plotted in this manner. (1) The results are normalized by the their value at $x/M = 15$ in order to be able to compare model predictions with experiments more easily. (2) The inverse value of the normalized

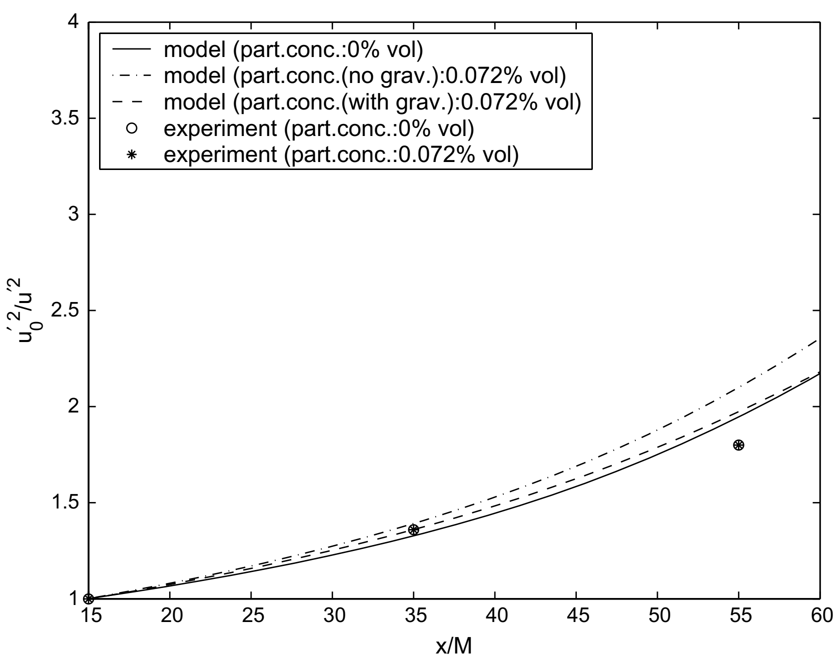

Fig. 1. Normalized inverse value of the turbulent kinetic energy as a function of the (dimensionless) distance behind the grid. Comparison between experimental data of Poelma et al. and model predictions for glass particles in water. The parameters at the start of the calculation have the following values $\psi = 0.00072$, $\phi = 0.0018$, $\delta = 0.0044$, $Re_f = 800$, $Fr = 0.0017$ and $\Lambda/L = 0.17$. $\phi\delta/Fr^2 = 2.74$.

turbulent kinetic energy is shown, as the normalized turbulent kinetic energy itself becomes rather small with increasing downstream distance. By using the inverse value differences between predictions and experiments can be seen more clearly. (3) The results for the flow with particles as well as for the flow without particles are plotted, so that the effect of the particles on the turbulent kinetic energy can be compared with the (normal) viscous decay of the turbulent energy in a decaying turbulent flow without particles. (4) The predictions for the cases with and without gravity effect are given, in order to separate the gravity effect from the inertia effect that the particles exert on the turbulence.

According to the model results shown in Fig. 1 there is a small damping effect of the particles on the turbulence because of their inertia and this effect is compensated by the small generation of turbulence due to the gravity effect. The experimental data also show no influence of the particles on the turbulence decay for this case.

In Fig. 2 a comparison is given between predictions and experimental data for the higher mass loading of $\phi = 0.0065$. From the case with particles but without gravity effect it can be seen again, that according to the model the inertia effect is not negligible. It causes a significant additional damping of the turbulence. When gravity is taken into account the inertia effect and the gravity effect nearly balance and the decay rate becomes similar to the one for the case without particles (like in Fig. 1). The agreement with experiments is reasonable. For this case we show in Fig. 3 the turbulence spectrum of the fluid as function of (dimensionless) wave number at the end of the simulation time. For the case with particles but without gravity effect damping occurs at all wave numbers. When the gravity effect is included there is a strong increase in the spectrum from the point in wave number space where the production occurs to larger wave numbers. The turbulence that is generated by the settling particles is transported to larger wave numbers (smaller eddies) by the cascade process of turbulence.

Finally we compare predictions with an experiment of Poelma et al. for glass particles with a diameter 509 μm and a mass loading of $\phi = 0.0092$. The result is given in Fig. 4. The calculation with particles but without gravity term show that the inertia effect causes an additional damping. However the calculation with particles and with gravity term shows, that the turbulence production due to the gravity effect is significantly

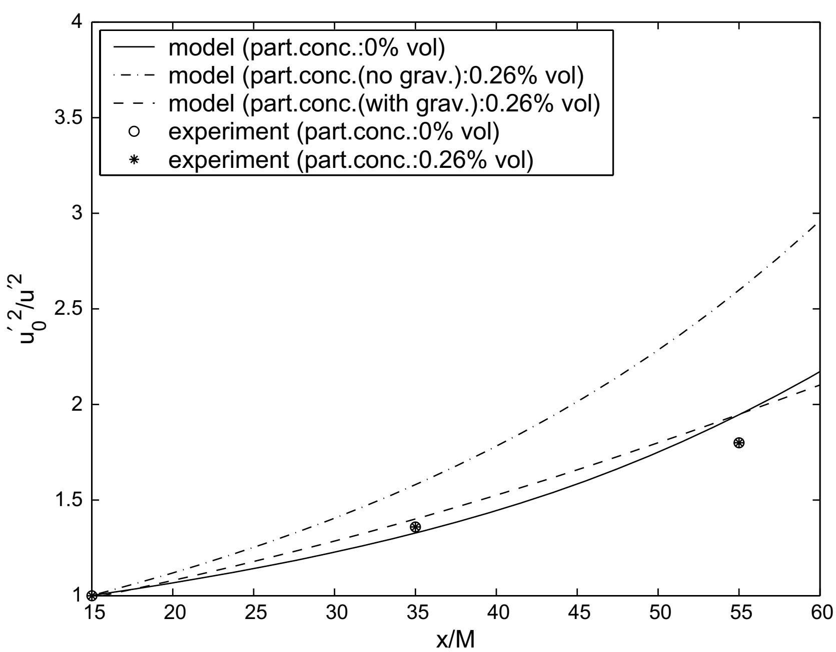

Fig. 2. Normalized inverse value of the turbulent kinetic energy as a function of the (dimensionless) distance behind the grid. Comparison between experimental data of Poelma et al. and model predictions for glass particles in water. The parameters at the start of the calculation have the following values $\psi = 0.0026$, $\phi = 0.0065$, $\delta = 0.0044$, $Re_f = 800$, $Fr = 0.0017$ and $\Lambda/L = 0.2$. $\phi\delta/Fr^2 = 9.9$.

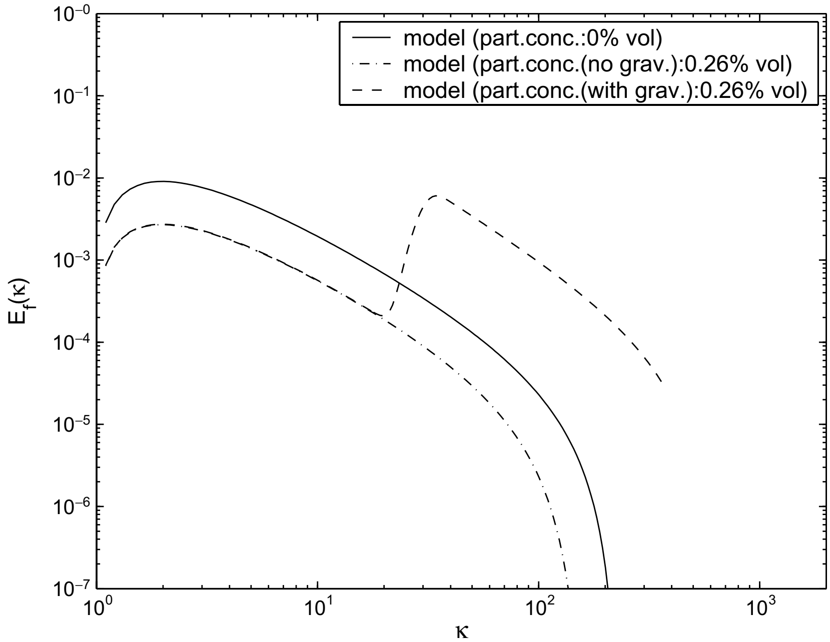

Fig. 3. Turbulent energy spectrum of the carrier fluid as a function of the (dimensionless) wave number. The parameters at the start of the calculation have the following values $\psi = 0.0026$, $\phi = 0.0065$, $\delta = 0.0044$, $Re_f = 800$, $Fr = 0.0017$ and $\Lambda/L = 0.2$. $\phi\delta/Fr^2 = 9.9$.

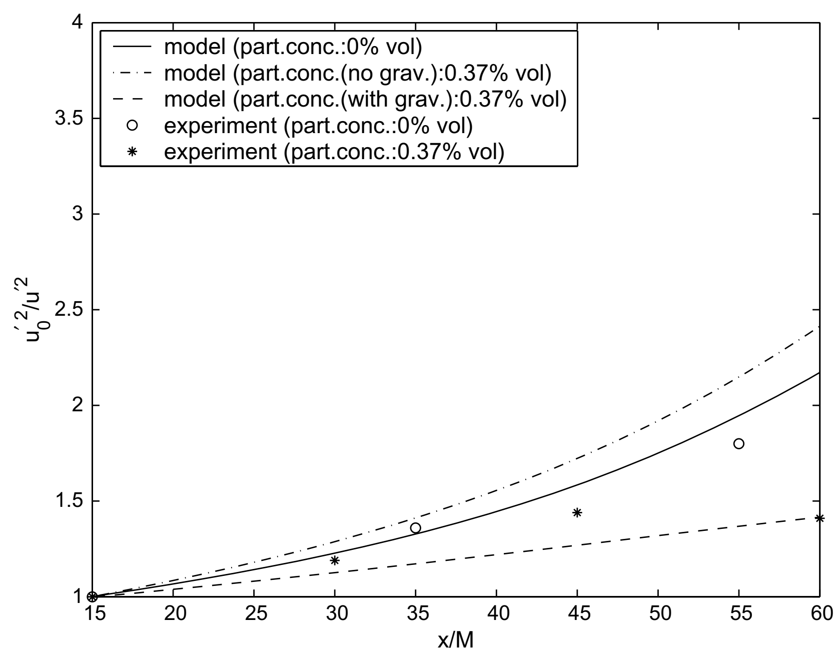

Fig. 4. Normalized inverse value of the turbulent kinetic energy as a function of the (dimensionless) distance behind the grid. Comparison between experimental data of Poelma et al. and model predictions for glass particles in water. The parameters at the start of the calculation have the following values $\psi = 0.0037$, $\phi = 0.0092$, $\delta = 0.0176$, $Re_f = 800$, $Fr = 0.0017$ and $\Lambda/L = 0.25$. $\phi\delta/Fr^2 = 56$.

stronger. The net result is that overall damping of turbulence for the case with particles is less than for the case without particles. This is in agreement with the experimental data.

Geiss et al. reported the results about the influence of particles on turbulence derived from wind tunnel experiments. The particle sizes were 120, 240 and 480 μm. In Fig. 5 we compare model predictions with experimental data for the 120 μm particles. The conclusion is that the inertia effect is rather small and more or less compensated by the gravity effect, so that the total effect of the particles on the turbulence is negligible in agreement with experiments.

In Fig. 6 a comparison is given for the 480 μm particles and a mass loading of $\phi = 0.8$. In this case the inertia effect is again rather small (as can be seen from the case with particles but without gravity effect). However the gravity effect is considerable. The experimental data in Fig. 6 show also a large effect due to turbulence production by settling of the particles, as the decay rate is much smaller than for the case without particles. The agreement between model predictions and experiments is good.

Schreck and Kleis studied the two-way coupling effect in grid-generated turbulence in a vertical water tunnel. They used two types of particles with a diameter of 650 μm: plastic particles and glass particles, with a relative density with respect to the water of 1.045 and 2.400 respectively. In Fig. 7 the comparison between an experiment with plastic particles and model predictions is shown. As can be seen from Fig. 7 there is a small additional damping of the turbulence due to the particles compared to the (normal) decay for the case of a turbulent flow without particles. The agreement between model predictions and experimental data for this additional damping effect is reasonable. The influence of the gravity effect is negligible, as could be expected from the very small value of the parameter $\phi\delta/Fr^2$.

In Fig. 8 the comparison between an experiment (again taken from Schreck and Kleis) with glass particles and model predictions is shown. When the gravity effect is not taken into account, the particles have an additional damping on the turbulence as also observed during the experiment. However the gravity effect is stronger, so that the overall result is that the turbulent kinetic energy for the flow with particles decreases less than the flow without particles. As can be seen from the figure this is not in agreement with the experimental data. We have no explanation for the disagreement between theory and experiment for this case.

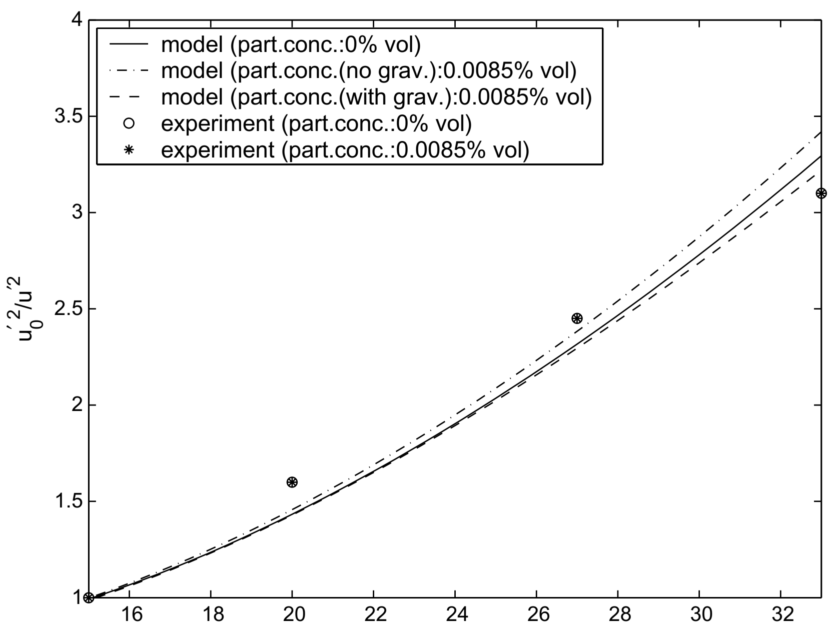

Fig. 5. Normalized inverse value of the turbulent kinetic energy as a function of the (dimensionless) distance behind the grid. Comparison between experimental data of Geiss et al. and model predictions for glass particles in air. The parameters at the start of the calculation have the following values $\psi = 0.000085$, $\phi = 0.166$, $\delta = 1.8$, $Re_f = 1060$, $Fr = 1.2$ and $\Lambda/L = 0.27$. $\phi\delta/Fr^2 = 0.2$.

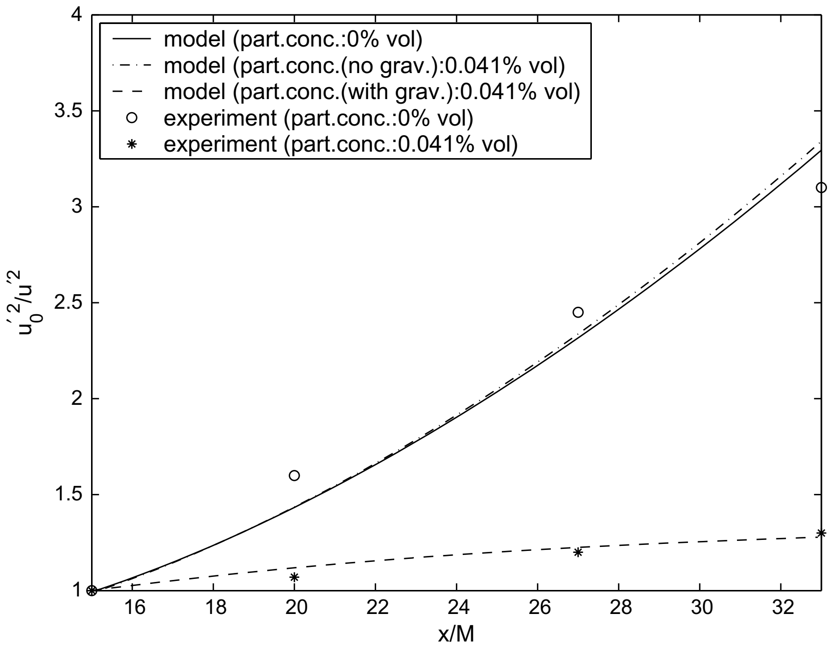

Fig. 6. Normalized inverse value of the turbulent kinetic energy as a function of the (dimensionless) distance behind the grid. Comparison between experimental data of Geiss et al. and model predictions for glass particles in air. The parameters at the start of the calculation have the following values $\psi = 0.00041$, $\phi = 0.8$, $\delta = 29.4$, $Re_f = 1060$, $Fr = 1.2$ and $\Lambda/L = 0.72$. $\phi\delta/Fr^2 = 16.5$.

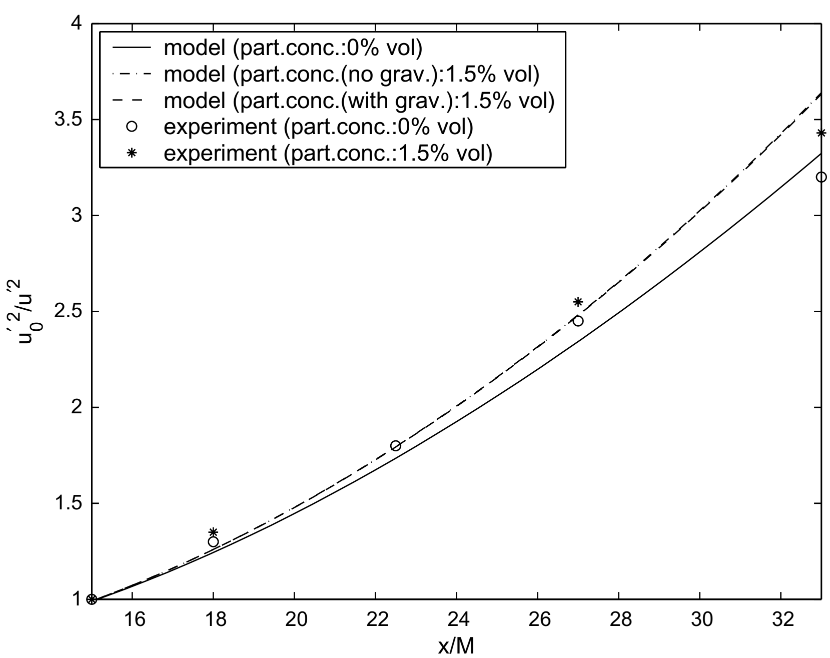

Fig. 7. Normalized inverse value of the turbulent kinetic energy as a function of the (dimensionless) distance behind the grid. Comparison between experimental data of Schreck and Kleis and model predictions for plastic particles in water. The parameters at the start of the calculation have the following values $\psi = 0.0150$, $\phi = 0.0157$, $\delta = 0.036$, $Re_f = 7000$, $Fr = 0.33$ and $\Lambda/L = 0.2$. $\phi\delta/Fr^2 = 0.0051$. Model predictions (for the case with particles) with or without gravity effect are almost identical.

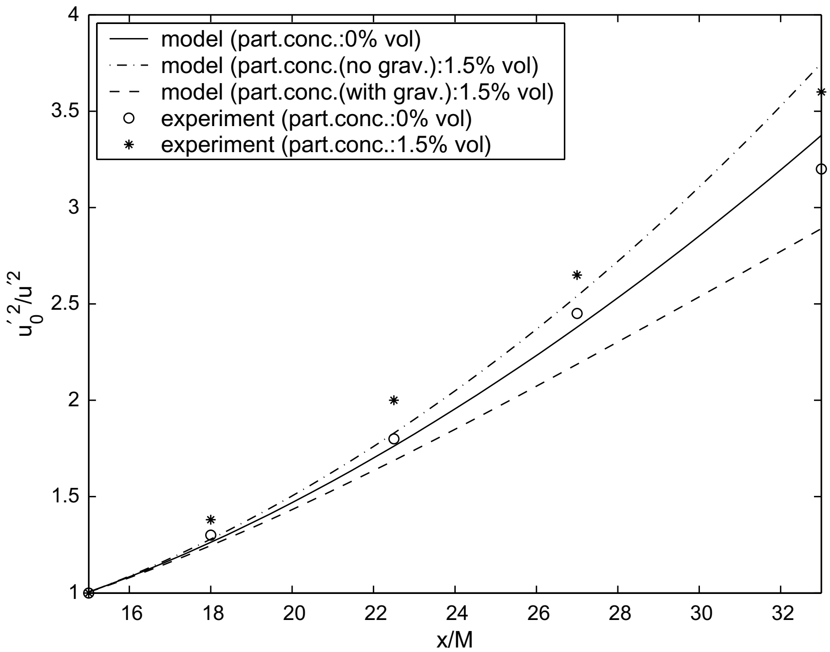

Fig. 8. Normalized inverse value of the turbulent kinetic energy as a function of the (dimensionless) distance behind the grid. Comparison between experimental data of Schreck and Kleis and model predictions for glass particles in water. The parameters at the start of the calculation have the following values $\psi = 0.015$, $\phi = 0.036$, $\delta = 0.070$, $Re_f = 7000$, $Fr = 0.024$ and $\Lambda/L = 0.08$. $\phi\delta/Fr^2 = 4.3$.

4. Discussion

It is important to be able to make a theoretical interpretation of experiments and numerical simulations of particle-laden turbulent flows and in particular of the two-way coupling effect. The theoretical model used in this publication is a one-fluid model for a homogeneous turbulent flow with particles. Ooms and Poelma (2004) compared predictions made with the model with results of direct numerical simulations for a decaying turbulent flow with particles. In this publication we have compared the predictions of the extended model with experiments in a water tunnel and in a wind tunnel. The agreement is reasonable. We think that our model is in particular suited to help making an interpretation of experiments in terms of the influence of the inertia effect and the gravity effect of the particles.

The gravity effect is determined by $\phi\delta/Fr^2$ and Λ/L. $\phi\delta/Fr^2$ indicates the strength of the turbulence production by particle settling with respect to the turbulence generated by the grid at the entrance to the tunnel. Λ/L indicates the position in wave number space where the turbulence production takes place. Both dimensionless groups are of importance for the influence of the gravity effect on the turbulence decay. According to our calculations the turbulent energy spectrum of the carrier fluid with particles shows a strong deviation from the spectrum of fluid without particles in case the gravity effect is strong (see Fig. 3). It would be very interesting to check this prediction experimentally.

We point out that in the derivation of our model it is assumed that the particle density is significantly larger than the density of the fluid. Otherwise Stokes' law for the particle motion is not valid, as the Bassett history force and the virtual mass need to be accounted for if the particle density is comparable to the fluid density. It is our intention to extend the theoretical model in that direction. As the number of publications dealing with accurate experiments concerning the two-way coupling effect in a homogeneous particle-laden flow is very limited, we made the comparison with the experiments of Poelma et al. (with glass particles) and Schreck and Kleis (with glass and plastic particles) although in their cases the particle density is not significantly larger than the fluid density.

In spite of the simplifications we feel that our theoretical model is helpful for explaining many features of the measured effects of the two-way coupling effect in a decaying particle-laden turbulent flow.

Appendix

To solve (7) the following integral length scale (L at $t = 0$) related parameters $\kappa = kL$, $\epsilon_L = \epsilon(1/L)$, $\gamma_L = \gamma(1/L)$ and $\rho_L = \rho_{\text{eff}}(1/L)$, and the following dimensionless functions $\epsilon_\kappa = \epsilon/\epsilon_L$, $\gamma_\kappa = \gamma/\gamma_L$ and $\rho_\kappa = \rho/\rho_L$ are introduced. Using the closure relations and the dimensionless functions defined earlier, the budget equation for the dimensionless energy flux ϵ_κ for the case of a decaying ($W(t,k) = 0$) turbulent particle-laden flow can be derived, including the turbulence production due to particle settling. Some lengthy but straightforward calculations result in the following expression

$$f(\tau,\kappa)\frac{\partial\epsilon_\kappa(\tau,\kappa)}{\partial\tau} + \frac{\partial\epsilon_\kappa(\tau,\kappa)}{\partial\kappa} + g(\tau,\kappa) = G_\kappa(\kappa), \tag{A.1}$$

where

$$f(\tau,\kappa) = \frac{1}{3}C_1\kappa^{-5/3}\rho_\kappa^{1/3}\epsilon_\kappa^{-1/3}\left[1 - \frac{1}{2}\frac{\epsilon_\kappa}{\rho_\kappa}\frac{(\partial\Phi/\partial\epsilon_\kappa)}{(\partial\Phi/\partial\rho_\kappa)}\right] \tag{A.2}$$

and

$$g(\tau,\kappa) = \frac{C_1}{Re_s}\left(\frac{\kappa\epsilon_\kappa^2}{\rho_\kappa^2}\right)^{1/3} + \left(\frac{C_1}{Re_s} + C\right)T_\kappa\left(\frac{\epsilon_\kappa^2\rho_\kappa}{\kappa^5}\right)^{1/3}, \tag{A.3}$$

with

$$\frac{\partial\Phi}{\partial\epsilon_\kappa} = C_3\phi\left[2\delta\{\delta\gamma_\kappa + (\delta\gamma_\kappa)^2\}\frac{\partial\gamma_\kappa}{\partial\epsilon_\kappa}\right]\Big/(1+\delta\gamma_\kappa)^4 \tag{A.4}$$

and

$$\frac{\partial\Phi}{\partial\rho_\kappa} = 1 + C_3\phi\left[2\delta\{\delta\gamma_\kappa + (\delta\gamma_\kappa)^2\}\frac{\partial\gamma_\kappa}{\partial\rho_\kappa}\right]\Big/(1+\delta\gamma_\kappa)^4 \tag{A.5}$$

in which

$$\frac{\partial\gamma_\kappa}{\partial\epsilon_\kappa} = \left[\frac{1}{3}\frac{\kappa^{2/3}}{\epsilon_\kappa^{2/3}\rho_\kappa^{1/3}}\right]\Big/\left[\frac{1}{C_4} + 1\right] \tag{A.6}$$

and

$$\frac{\partial\gamma_\kappa}{\partial\rho_\kappa} = \left[-\frac{\kappa^2}{C_4\rho_\kappa^2} - \frac{1}{3}\frac{\epsilon_\kappa^{1/3}\kappa^{2/3}}{\rho_\kappa^{4/3}}\right]\Big/\left[\frac{1}{C_4} + 1\right]. \tag{A.7}$$

T_κ is given by

$$T_\kappa = \frac{(\phi/\delta)[1 + \delta\gamma_\kappa - 2\{\delta\gamma_\kappa\}^2]/[2 - 2\{\delta\gamma_\kappa\}^2]}{1 - \psi + \phi[1 - \delta\gamma_\kappa]/[2 - 2\{\delta\gamma_\kappa\}^2]}, \tag{A.8}$$

and G_κ by

$$G_\kappa = \frac{\phi\delta}{[1 + \phi(1+2\delta)/(1+\delta)^2]}\frac{1}{Fr^2}\left[\frac{1}{\sigma L(2\pi)^{1/2}}exp\left(\frac{-\frac{1}{2}(\kappa - 2\pi L/\Lambda)^2}{(\sigma L)^2}\right)\right]. \tag{A.9}$$

In the above equations τ is the dimensionless time defined as $\tau = t/\tau_c$ with $\tau_c = L/u_L$. u_L the macroscopic velocity scale at $t = 0$. $\delta = \tau_p\gamma_L$ is the dimensionless particle response time. The suspension Reynolds number is defined by $Re_s = Lu_L/v_L$. v_L is the effective kinematic viscosity of the suspension for $k = L^{-1}$ and is given by $v_L = v(\rho_f/\rho_L)$ with ρ_L the effective density of the suspension for $k = L^{-1}$ given by $\rho_L = \rho_f\ [1 - \psi + \phi(1+2\delta)/(1+\delta)^2]$. The fluid Reynolds number is defined by $Re_f = Lu_L/v$ and is related to the suspension Reynolds number in the following way $Re_f = Re_s v_L/v$. Fr is defined by $Fr = (\rho_p {u_L}^2)/(\Delta\rho g L)$ in which ρ_p is the mass density of the particles, $\Delta\rho$ the difference in mass density between the particles and the carrier fluid and g the acceleration due to gravity.

Λ/L is the ratio of the length scale Λ of the turbulence generated by the settling particles and the integral length scale L of the turbulence generated by the grid in a water tunnel or wind tunnel. As mentioned for Λ we choose an expression given by Mizukami et al. (1992). It is given by

$$\Lambda = C_{\mathrm{u}} \frac{v_{\mathrm{tv}}^3}{P} \left[\frac{Pd(\theta/d)^{2/3}}{v_{\mathrm{tv}}^3} \right], \tag{A.10}$$

in which v_{tv} is the settling velocity of the particles, $P = \phi v_{\mathrm{tv}}^2/\tau_{\mathrm{p}}$ the kinetic energy per unit mass generated by the settling particles, d the particle diameter and θ the wake momentum diameter defined as $\theta = (C_D d^2/8)^{1/2}$ with $C_D = 24(1 + Re_{\mathrm{tv}}^{2/3}/6)/Re_{\mathrm{tv}}$ and $Re_{\mathrm{tv}} = v_{\mathrm{tv}} d/\mu$. C_{u} is a constant with the value 54 in vertical direction and 14 in horizontal direction. As explained we follow in our theoretical model a suggestion by Batchelor for non-isotropic but homogeneous turbulence by averaging over all directions of wavenumber space. Based in this suggestion we took an average value of $C_{\mathrm{u}} = 34$ for our calculations.

$C = C_1 C_2 (\simeq 1)$ and the constants C_3 and C_4 are equal to $C_3 = [1 + \phi(1 + 2\delta)/(1 + \delta)^2]^{-1}$ and $C_4 = C_2 Re_{\mathrm{s}}$. The functions ρ_κ and γ_κ can be shown to be given by

$$\rho_\kappa = \left[1 - \psi + \phi \frac{1 + 2\delta\gamma_\kappa}{(1 + \delta\gamma_\kappa)^2}\right] \Bigg/ \left[1 - \psi + \phi \frac{1 + 2\delta}{(1 + \delta)^2}\right] \tag{A.11}$$

and

$$\gamma_\kappa = \left[\frac{\kappa^2}{C_2 Re_{\mathrm{s}} \rho_\kappa} + \frac{\epsilon_\kappa^{1/3} \kappa^{2/3}}{\rho_\kappa^{1/3}}\right] \Bigg/ \left[\frac{1}{C_4} + 1\right]. \tag{A.12}$$

The budget Eq. (A.1) for the energy flux is interesting. The g-term in the equation represents the energy dissipation due to the particle–fluid friction and the internal fluid friction. The G_κ-term represents, as mentioned, the turbulence production due to particle settling. The first two terms in (A.1) describe the influence of the cascade process of turbulence. When the g-term and G_κ-term are neglected (A.1) becomes a kind of wave equation in κ-space with a time- and wavenumber-dependent wave velocity $1/f(\tau, \kappa)$.

In order to be able to solve (A.1) an initial condition and a boundary condition are needed. We will study that part of the spectrum that runs from $\kappa = 1$ via the inertial subrange well into the dissipation range. We assume also that for $\tau < 0$ the stirring force is still feeding turbulent energy at $\kappa = 1$ into the turbulence spectrum. At $\tau = 0$ the stirring force is stopped, the energy flux at $\kappa = 1$ disappears and the decay process starts. So the boundary condition is: for $\tau \geqslant 0$ the energy flux $\epsilon_\kappa = 0$ at $\kappa = 1$. As initial condition ($\tau = 0$) we choose for the energy flux the following spectrum for $\kappa \geqslant 1$

$$\epsilon_\kappa = \left(1 - \frac{C_1}{4Re_{\mathrm{f}}} \kappa^{4/3}\right)^3 \Bigg/ \left(1 - \frac{C_1}{4Re_{\mathrm{f}}}\right)^3. \tag{A.13}$$

For values of κ considerably smaller than $(4Re_{\mathrm{f}}/C_1)^{3/4}$ this spectrum is approximately equal to $\epsilon_\kappa = 1$. So the (dimensionless) flux is for such κ-values equal to its value at $\kappa = 1$ (or $k = L^{-1}$). For larger κ-values the energy flux ϵ_κ decreases due to viscous dissipation. The spectrum of (A.13) is derived by L'vov et al. using certain approximations. We use this spectrum as our initial condition and calculate its development in time due to the decay process. According to the idea of universality of turbulence the properties of the energy flux though the eddies and the energy spectrum become independent of the initial condition after a relaxation time. This is due to the strong coupling between eddies of different size. So we do not expect a strong dependence of our results on the initial condition.

After ϵ_κ has been calculated for a certain case from (A.1) the dimensionless energy spectrum of the suspension can be determined using the closure relation (9)

$$E_{\mathrm{s}}(\tau, \kappa) = \left(\frac{\epsilon_\kappa^2 \rho_\kappa}{\kappa^5}\right)^{1/3}. \tag{A.14}$$

L'vov et al. have shown that the energy spectrum of the carrier fluid is given by

$$E_{\mathrm{f}}(\tau,\kappa)=\left(\frac{\epsilon_{\kappa}^{2}}{\rho_{\kappa}^{2}\kappa^{5}}\right)^{1/3}\left[\frac{1}{1-\psi+\phi(1+\delta)/(1+\delta)^{2}}\right]. \tag{A.15}$$

In this way it becomes possible to study the decay of the turbulent energy spectrum of the fluid as function of the relevant dimensionless groups, namely the particle volume fraction ψ, the particle mass fraction ϕ, the dimensionless particle response time δ, the fluid Reynolds number Re_{f}, the Froude number Fr, and the ratio Λ/L. The energy spectrum $E_{\mathrm{f}}(\tau,\kappa)$ yields after integration with respect to κ the turbulence kinetic energy of the carrier fluid as function of time, which in this publication is compared with experimental data. From (A.9) can be seen that the gravity effect is determined by the following two dimensionless groups: $\frac{\phi\delta}{[1+\phi(1+2\delta)/(1+\delta)^{2}]}\frac{1}{Fr^{2}}$ and Λ/L. For most applications ϕ is so small, that the value of the factor $[1+\phi(1+2\delta)/(1+\delta)^{2}]$ is close to unity and then the first dimensionless group reduces to $\phi\delta/Fr^{2}$. $\phi\delta/Fr^{2}$ indicates the strength of the turbulence generated by the particles due to their settling in the carrier fluid with respect to the (decaying) turbulence generated by the grid at the entrance to the tunnel. Λ/L indicates the position in wave number space where the turbulence production takes place. Eq. (A.1) together with its initial condition and boundary condition has been solved numerically. The computer code is available on request.

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